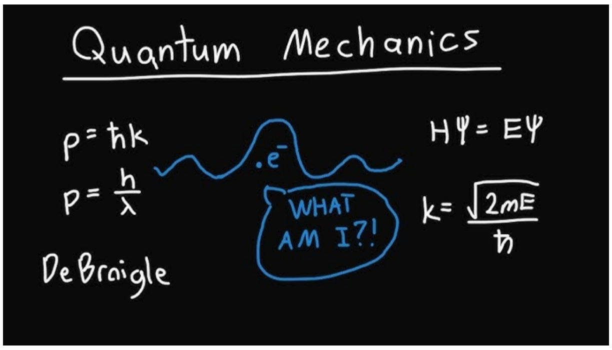

East African Institute for Fundamental Research (EAIFR)

Course:	**Quantum Mechanics I: basic concepts**
Instructor:	Andrea Marini
Affiliation:	National Research Council, Italy
E-mail:	andrea.marini@cnr.it

Course description: The goal of this course is to introduce the basic concepts of modern Quantum Mechanics.

The course is divided in Topics and each topic includes some pages of reference books and papers, hand written notes, video captures of the lectures and, eventually, assignments.

The idea is that the student takes notes during the lecture. These notes can be checked and improved or by using the lecture video *with the help of the hand written notes* and of the references provided. The student is asked to repeat each proof carefully and in detail by taking particular care to the formal aspects of the mathematical derivations.

All references, notes and assignment can be found on the Moodle page.

In the following you can find a detailed list of topics discussed during the lectures.

Part I

1. Fundamental concepts I

1.1 The Stern–Gerlach experiment.

1.1.1 The sequential Stern–Gerlach experiment.

1.2 The Dirac Notation.

1.2.1 Kets, bras and observables.

1.2.2 Operators and representations.

1.2.3 Eigenstates as base kets.

1.2.4 Matrix representations.

1.2.5 Spin $\frac{1}{2}$ systems.

References: Sakurai (1994), pages 1-23
Notes: Hand written L.1 (19 pages)

2. Fundamental concepts II

2.1 Quantum–Mechanics theory of measurements.

2.1.1 Definition of Measure in Quantum–Mechanics.

2.1.2 Spin $\frac{1}{2}$ systems: the Stern–Gerlach experiment revisited.

2.1.3 Pauli matrices.

2.1.4 Compatible observables.

2.1.5 Degenerate eigen–kets spaces.

2.1.6 Incompatible observables.

2.1.6 The uncertain relation.

2.2 Basis transformations.

2.2.1 Transformation matrix.

2.2.2 Diagonalization.

2.2.3 Unitary equivalent operator.

References: Sakurai (1994), pages 23-41
Notes: Hand written L.2 (20 pages)

3. Fundamental concepts III

3.1 The position operator.

3.2 Translations and the momentum operator.

3.2.1 Canonical commutation relations.

3.3 Wave functions.
3.4 Position and momentum basis.

References: Sakurai (1994), pages 41-60
Notes: Hand written L.3 (13 pages)

4. Schrödinger and Heisenberg equations

4.1 The time evolution operator.
4.2 The Schrödinger equation.

4.2.1 Energy eigenkets.

4.2.2 Time dependence of expectation values.

4.2.3 Spin precession.

4.2.4 Correlation amplitude and energy uncertain relation.

4.3 The Heisenberg picture.

4.3.1 The Ehrenfest theorem.

4.3.2 Base kets time evolution.

References: Sakurai (1994), pages 68-89
Notes: Hand written L.4 (26 pages)

5. One–dimensional Harmonic oscillator and Schrödinger wave–equation

5.1 The one–dimensional Harmonic oscillator.

5.1.1 Creation and annihilation operators.

5.1.2 Eigefunctions: Hermite polynomials.

5.1.3 Time evolution.

5.1.4 Coherent states.

5.2 Time–dependent and static Schrödinger wave–equation.

5.2.1 Physical interpretation of the wave–function.

5.2.2 Continuity equation.

References: Sakurai (1994), pages 89-103
Notes: Hand written L.5 (20 pages)

6. Schrödinger wave–equation in paradigmatic Hamiltonian systems

6.1 General properties of motion in one dimension.
6.2 The potential well.
6.3 Motion in a homogeneous field.

References: Landau und Lifshitz (1965), pages 60-80
Notes: Hand written L.6 (11 pages)

7. The classical limit

7.1 The $\hbar \to 0$ limit.
7.2 The semi–classical Wentzel–Kramers and Brillouin (WKB) solution of Schrödinger wave–equation.
7.3 Boundary conditions in the quasi–classical case.
7.4 Bohr and Sommerfield quantisation rule.
7.5 Choerences and fluctuations (a perspective on Many–Body Perturbation Theory).

References: Sakurai (1994), pages 103-106
Landau und Lifshitz (1965), pages 158-167
Notes: Hand written L.7 (16 pages)

8. Propagators and path integrals

8.1 Propagators in wave–mechanics.

8.1.2 Properties of the propagators.

8.1.3 Propagators as Green's functions solvers of differential equations.

8.1.4 Example of Propagator: free particle.

8.1.5 Example of Propagator: the harmonic oscillator.

8.1.5.1 Trace of the harmonic oscillator propagator and link to the statistical partition function.

8.1.5.2 Fourier–Laplace transform of the harmonic oscillator propagator and spectral properties.

8.2 Feynman's formulation of quantum mechanics.

8.2.1 Propagator as transition amplitude.

8.2.2 Path integrals: quantum versus classical definition.

8.2.3 Feynman's formulation.

References: Sakurai (1994), pages 109-123
Notes: Hand written L.8 (28 pages)

9. Potentials and Gauge transformations

9.1 Potentials and phases.
9.2 Quantum mechanical formulation of Lorentz force.
9.3 Continuity equation in the presence of Electromagnetic fields.
9.4 Gauge transformations.
9.5 The Aharanov–Bohm effect.

9.5.1 Cylinder geometry.

9.5.2 Feynman's path integrals approach and magnetic flux quantization.

9.A Stoke's theorem.

References: Sakurai (1994), pages 123-139
Zaric u. a. (2004)
Notes: Hand written L.9 (19 pages)

10. Rotations and angular momentum quantization

10.1 Finite and infinitesimal Cartesian rotations.
10.2 Quantum mechanical formulation of rotations.
10.3 Spin $\frac{1}{2}$ rotations.

10.3.1 Representation of rotations in the $|\pm\rangle$ space.

10.4 Eigenstates and eigenvalues of the angular momentum.

10.4.1 Commutation relations and ladder operators.

10.4.2 $\hat{J}^2$ and $\hat{J}_z$ eigenvalues.

10.4.3 Matrix elements of the angular momentum operators.

10.A Reprint of Werner u. a. (1975).

References: Sakurai (1994), pages 152-168
Sakurai (1994), pages 187-192
Werner u. a. (1975)
Notes: Hand written L.10 (20 pages)

11. Orbital angular momentum

11.1 Orbital angular momentum.

11.1.1 Orbital angular momentum as rotations generator.

11.1.2 Spherical harmonics.

11.2 Angular momentum composition.

11.2.1 Introduction: orbital plus spin and spin plus spin.

11.2.2 Formal theory.

11.3 Glebsch–Gordan coefficients

11.3.1 Recursion formula.

11.3.2 Orbital plus spin case.

11.3.3 Spin plus spin case.

References: Sakurai (1994), pages 195-215
Notes: Hand written L.11 (30 pages)

12. Representations of spatial rotations and Schwinger model

12.1 Representations of spatial rotations.
12.2 Euler angles.

12.2.1 Spin $\frac{1}{2}$ space.

12.2.2 General $\hat{\vec{J}}$ case.

12.3 Schwinger model of angular momentum.

References:	Sakurai (1994), pages 171-174
	Sakurai (1994), pages 217-221
Notes:	Hand written L.12 (12 pages)

13. Brief introduction to quantum statistical mechanics

13.1 Polarized versus mixed beams: mixed versus pure states.

13.2 Ensemble average and density operator.

13.2.1 Example Nr.1: a polarized $\hat{S}_z$ beam.

13.2.2 Example Nr.2: a polarized $\hat{S}_c$ beam.

13.2.3 Example Nr.3: an unpolarized spin beam.

13.2.3 Example Nr.3: a partially polarized spin beam.

13.3 Ensembles time evolution.

13.3 Quantum statistical mechanics

References:	Sakurai (1994), pages 174-187
Notes:	Hand written L.13 (11 pages)

References

[Landau und Lifshitz 1965] Landau, L. D. ; Lifshitz, E. M.: *Quantum Mechanics, Non Relativistic Theory. Volume 3*. 1965

[Sakurai 1994] Sakurai, J.J. ; Tuan, S.F. (Hrsg.): *Modern Quantum Mechanics, Revised Edition*. Addison-Wesley, 1994

[Werner u. a. 1975] Werner, S. A. ; Colella, R. ; Overhauser, A. W. ; Eagen, C. F.: Observation of the Phase Shift of a Neutron Due to Precession in a Magnetic Field. In: *Phys. Rev. Lett.* 35 (1975), S. 1053–1055. – URL http://doi.org/10.1103/PhysRevLett.35.1053

[Zaric u. a. 2004] Zaric, S. ; Ostojic, G. N. ; Kono, J. ; Shaver, J. ; Moore, V. C. ; Strano, M. S. ; Hauge, R. H. ; Smalley, R. E. ; Wei, X.: Optical signatures of the Aharonov-Bohm phase in single-walled carbon nanotubes. In: *Science* 304 (2004), Nr. 5674, S. 1129–1131. – URL https://www.science.org/doi/10.1126/science.1096524