



East African Institute for Fundamental Research (EAIFR)

Course: Quantum Mechanics I: basic concepts
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Course description: The goal of this course is to introduce the basic concepts of modern Quantum Mechanics.

The course is divided in Topics and each topic include some pages of reference books, hand written notes, video captures of the lectures and, eventually, assignments.

The idea is that the student takes notes during the lecture and/or by using the video and study them in detail *with the help of the hand written notes* and of the references provided. The student is asked to repeat each proof carefully and in detail by taking particular care to the formal aspects of the mathematical derivations.

All references, notes and assignment can be found on the [Moodle page](#).

In the following you can find a detailed list of topics discussed during the lectures. For each topic it is reported the corresponding video. For example [Video_2.3] means the Video #3 of the lecture #2.

Part I ---

1. Fundamental concepts I

- 1.1 The Stern–Gerlach experiment.
 - 1.1.1 The sequential Stern–Gerlach experiment.
- 1.2 The Dirac Notation.
 - 1.2.1 Kets, bras and observables.
 - 1.2.2 Operators and representations.
 - 1.2.3 Eigenstates as base kets.
 - 1.2.4 Matrix representations.
 - 1.2.5 Spin $\frac{1}{2}$ systems.

References: [Sakurai \(1994\)](#), pages 1-23
Notes: Hand written L.1 (19 pages)

2. Fundamental concepts II

- 2.1 Quantum–Mechanics theory of measurements.
 - 2.1.1 Definition of Measure in Quantum–Mechanics.
 - 2.1.2 Spin $\frac{1}{2}$ systems: the Stern–Gerlach experiment revisited.
 - 2.1.3 Pauli matrices.
 - 2.1.4 Compatible observables.
 - 2.1.5 Degenerate eigen–kets spaces.
 - 2.1.6 Incompatible observables.
 - 2.1.6 The uncertain relation.
- 2.2 Basis transformations.
 - 2.2.1 Transformation matrix.
 - 2.2.2 Diagonalization.
 - 2.2.3 Unitary equivalent operator.

References: [Sakurai \(1994\)](#), pages 23-41
Notes: Hand written L.2 (20 pages)

3. Fundamental concepts III

- 3.1 The position operator.
- 3.2 Translations and the momentum operator.
 - 3.2.1 Canonical commutation relations.

3.3 Wave functions.

3.4 Position and momentum basis.

References: [Sakurai \(1994\)](#), pages 41-60

Notes: Hand written L.3 (13 pages)

4. Schrödinger and Heisenberg equations

4.1 The time evolution operator.

4.2 The Schrödinger equation.

4.2.1 Energy eigenkets.

4.2.2 Time dependence of expectation values.

4.2.3 Spin precession.

4.2.4 Correlation amplitude and energy uncertain relation.

4.3 The Heisenberg picture.

4.3.1 The Ehrenfest theorem.

4.3.2 Base kets time evolution.

References: [Sakurai \(1994\)](#), pages 68-89

Notes: Hand written L.4 (26 pages)

5. One-dimensional Harmonic oscillator and Schrödinger wave-equation

5.1 The one-dimensional Harmonic oscillator.

5.1.1 Creation and annihilation operators.

5.1.2 Eigenfunctions: Hermite polynomials.

5.1.3 Time evolution.

5.1.4 Coherent states.

5.2 Time-dependent and static Schrödinger wave-equation.

5.2.1 Physical interpretation of the wave-function.

5.2.2 Continuity equation.

References: [Sakurai \(1994\)](#), pages 89-103

Notes: Hand written L.5 (20 pages)

6. Schrödinger wave-equation in paradigmatic Hamiltonian systems

6.1 General properties of motion in one dimension.

6.2 The potential well.

6.3 Motion in a homogeneous field.

References: [Landau und Lifshitz \(1965\)](#), pages 60-80

Notes: Hand written L.6 (11 pages)

7. The classical limit

- 7.1 The $\hbar \rightarrow 0$ limit.
- 7.2 The semi-classical Wentzel-Kramers and Brillouin (WKB) solution of Schrödinger wave-equation.
- 7.3 Boundary conditions in the quasi-classical case.
- 7.4 Bohr and Sommerfeld quantisation rule.
- 7.5 Coherences and fluctuations (a perspective on Many-Body Perturbation Theory).

References: [Sakurai \(1994\)](#), pages 103-106
[Landau und Lifshitz \(1965\)](#), pages 158-167

Notes: Hand written L.7 (16 pages)

8. Propagators and path integrals

- 8.1 Propagators in wave-mechanics.
 - 8.1.2 Properties of the propagators.
 - 8.1.3 Propagators as Green's functions solvers of differential equations.
 - 8.1.4 Example of Propagator: free particle.
 - 8.1.5 Example of Propagator: the harmonic oscillator.
 - 8.1.5.1 Trace of the harmonic oscillator propagator and link to the statistical partition function.
 - 8.1.5.2 Fourier-Laplace transform of the harmonic oscillator propagator and spectral properties.
- 8.2 Feynman's formulation of quantum mechanics.
 - 8.2.1 Propagator as transition amplitude.
 - 8.2.2 Path integrals: quantum versus classical definition.
 - 8.2.3 Feynman's formulation.

References: [Sakurai \(1994\)](#), pages 109-123
[Zaric u. a. \(2004\)](#)

Notes: Hand written L.8 (28 pages)

9. Potentials and Gauge transformations

- 9.1 Potentials and phases.
- 9.2 Quantum mechanical formulation of Lorentz force.
- 9.3 Continuity equation in the presence of Electromagnetic fields.
- 9.4 Gauge transformations.
- 9.5 The Aharanov-Bohm effect.
 - 9.5.1 Cylinder geometry.
 - 9.5.2 Feynman's path integrals approach and magnetic flux quantization.

9.A Stoke's theorem.

9.B Reprint of [Zaric u. a. \(2004\)](#).

References: [Sakurai \(1994\)](#), pages 123-139
[Zaric u. a. \(2004\)](#)

Notes: Hand written L.9 (19 pages)

10. Rotations and angular momentum quantization

10.1 Finite and infinitesimal Cartesian rotations.

10.2 Quantum mechanical formulation of rotations.

10.3 Spin $\frac{1}{2}$ rotations.

10.3.1 Representation of rotations in the $|\pm\rangle$ space.

10.4 Eigenstates and eigenvalues of the angular momentum.

10.4.1 Commutation relations and ladder operators.

10.4.2 \hat{J}^2 and \hat{J}_z eigenvalues.

10.4.3 Matrix elements of the angular momentum operators.

10.A Reprint of [Werner u. a. \(1975\)](#).

References: [Sakurai \(1994\)](#), pages 152-168
[Sakurai \(1994\)](#), pages 187-192
[Werner u. a. \(1975\)](#)

Notes: Hand written L.10 (20 pages)

11. Orbital angular momentum

11.1 Orbital angular momentum.

11.1.1

References: [Sakurai \(1994\)](#), pages 187-XXX
Notes: Hand written L.11 (XX pages)

11. Ensembles quantum mechanics

11.1 A

11.1.1 B

11.2 C

References: [Sakurai \(1994\)](#), pages 174-187
Notes: Hand written L.11 (XX pages)

References

- [Landau und Lifshitz 1965] LANDAU, L. D. ; LIFSHITZ, E. M.: *Quantum Mechanics, Non Relativistic Theory. Volume 3.* 1965
- [Sakurai 1994] SAKURAI, J.J. ; TUAN, S.F. (Hrsg.): *Modern Quantum Mechanics, Revised Edition.* Addison-Wesley, 1994
- [Werner u. a. 1975] WERNER, S. A. ; COLELLA, R. ; OVERHAUSER, A. W. ; EAGEN, C. F.: Observation of the Phase Shift of a Neutron Due to Precession in a Magnetic Field. In: *Phys. Rev. Lett.* 35 (1975), S. 1053–1055. – URL <http://doi.org/10.1103/PhysRevLett.35.1053>
- [Zaric u. a. 2004] ZARIC, S. ; OSTOJIC, G. N. ; KONO, J. ; SHAVER, J. ; MOORE, V. C. ; STRANO, M. S. ; HAUGE, R. H. ; SMALLEY, R. E. ; WEI, X.: Optical signatures of the Aharonov-Bohm phase in single-walled carbon nanotubes. In: *Science* 304 (2004), Nr. 5674, S. 1129–1131. – URL <https://www.science.org/doi/10.1126/science.1096524>